Professor Leishman's assertion that "in the helicopter rotor blade the induced pressure distribution on the wing is no longer given in the form of Eq. (2) of Ref. 1" is very true. But this assertion doesn't alter our use of Eq. (2) in Ref. 1. As mentioned, the leading edge suction model can use lifting-line theory in which Eqs. (1-6) in Ref. 1 are accurate. In this case, any special numerical extrapolation need not be employed.

We thank Professor Leishman for reminding us that "the induced drag is certainly affected by unsteady effects," but his assertion that our statements were to the contrary is not true. We have not concluded that the induced drag is not affected by unsteady effects. The purpose of developing the leading-edge suction model is to include unsteady effects completely. We would like to know how to use the equivalent apparent mass contributions to the unsteady induced drag. Professor Leishman has also reminded us of the viscosity effects and his formula

$$Cd = 2\pi/\beta\ (1\text{-}\epsilon)\ \alpha^2$$

But again, this was beyond the scope of Ref. 1.

References

[1]Li, C. E., Poling, D. R., and Wu, D., "Induced Drag Based on Leading Edge Suction for a Helicopter in Forward Flight," *AIAA Journal*, Vol. 28, No. 2, 1990, pp. 201-204.

[2]Egolf, T. A., and Landgrebe, A. J., "Helicopter Rotor Wake Geometry and Its Influence on Forward Flight," NASA CR 3726, Dec. 1983.

[3]Wang, S., and Xu, Z., "A Simplified Method for Predicting Rotor Blade Airloads," *Acta Aeronautica et Aeronautic Sinica*, Vol. 3, No. 2, 1983, pp. 1-17.

[4]Sadler, S. G., "Development and Application of a Method for Predicting Rotor Free Wake Positions and Resulting Rotor Blade Air Loads," NASA CR 1911, Dec. 1971.

[5]Li, C. E., "On The Unsteady Leading Edge Suction of a Swept-back Wing," *Acta Aerodynamica Sinica,* Vol. 7, No. 1, 1989, pp. 1-11.

Comment on "Calculation of Asymmetric Vortex Separation on Cones and Tangent Ogives Based on a Discrete Vortex Model"

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CHIN et al.[1] used the two-vortex model and suggested, as done earlier by Dyer et al.,[2] that the multiple solutions provide an alternative explanation of the existence of asymmetric vortex separation at zero sideslip and the measured side-force can only be predicted by second-branch solutions.

Vortex characteristics of bodies of revolution have been studied extensively through the use of several methods, including the vortex-element methods, in the framework of slender body theory. Also with the added predicaments of where are the separation lines, what is the rate of shedding of vorticity, how does the separating stream surface leave the body, what conditions should be satisfied at the separation line(s), and what is the relationship between the Kutta condition and separation from a smooth surface. Chin et al.'s technical note does not deal with these questions and confines itself to the use of the modified versions of Bryson's[3] and Dyer et al.'s[2] two-vortex models, with additional ad hoc assumptions.

Each vortex is connected to the fixed and prescribed separation points with a cut (a feeding sheet) of vanishing small vorticity. The net force on the total vortex system is rendered globally zero, separately on each side. The line vortex does not lie along a streamline, or in the cross flow. Its velocity is not equal to the local fluid velocity. The moment acting on each vortex and its feeding sheet is not zero and cannot be rendered zero without introducing additional ad hoc assumptions. Furthermore, the model ignores the effect of the secondary separations.

Bryson forced the separation line and the positions of the line vortices to be symmetric about the incidence plane. A Kutta-type condition (the tangential component of the cross-flow is zero on the body) was invoked at the separation points. In spite of its remarkable simplicity, Bryson's analysis predicted the normal force at the early stages of motion fairly accurately.[4] At later times, the force drops sharply and unrealistically. However, the most remarkable feature of Bryson's model is that it owes its limited success to the forced symmetry of the vortices.

In 1969, Davis[5] recast Bryson's model to remove the forced symmetry. The use of initial values including symmetric separation points exactly identical to those of Bryson failed to produce symmetric vortex positions. It was discovered that the matrix yielding the rates of change of the strengths and positions of the vortices is ill-conditioned and the slightest truncation error leads to abnormally large values for the vortex strengths and positions. The unexpected and surprising asymmetry of the vortices was not interpreted as an explanation of the sectional side force which has since become an important problem.[6] Rather, it was discovered that the two force-balance equations for the vortex-cut are the source of the ill-conditioned behavior of the matrix yielding the strengths and positions of the two vortices. This, in turn, is due to the fact that the moment acting on a vortex and its connecting sheet is not zero. In other words, imposed symmetry can hide computational instability. Thus, the vortex asymmetry resulting from the ill-conditioned nature of the approximate equations explains neither the existence nor the non-deterministic behavior of the side forces. This is not to say that the side force is not a consequence of vortex asymmetry, but rather to emphasize that the source of asymmetry resides upstream of the separation points, not in the unstable behavior of the approximate equations, based on numerous ad hoc assumptions and empirical parameters. In fact, a careful multivortex analysis of the impulsively-started flow by Sarpkaya and Shoaff,[7] without resorting to the vortex-cut and no-force assumption, did not yield a bifurcation to asymmetric state, at least without introducing an asymmetry in the separation points and/or in the shear layers. A similar conclusion has been reached by Almosnino[8] using a non-linear vortex-lattice method. A detailed discussion of the vortex methods is given by Sarpkaya.[9] A thorough discussion of the forebody and missile side forces is given by Hall.[6]

It is concluded that the two-vortex model is a crude approximation to a very complex problem, the use of force-free feeding-sheets connecting the prescribed separation points to the line vortices does not yield a moment-free system, the resulting equations are ill-conditioned for the vortex strengths and positions, and the vortex asymmetry resulting from the ill-conditioned nature of the governing set of equations explains neither the existence nor the nondeterministic behavior of the side forces, with or without circulation reduction and a number of ad hoc assumptions to bring the calculated and measured values into closer agreement.

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References

[1]Chin, S., Lan, C. E., and Gainer, T. G., "Calculation of Asymmetric Vortex Separation on Cones and Tangent Ogives Based on a Discrete Vortex Model," *AIAA Journal*, Vol. 27, No. 12, 1989, pp. 1824–1826.

[2]Dyer, D. E., Fiddes, S. P., and Smith, J. H. B., "Asymmetric Vortex Formation from Cones at Incidence—A Simple Inviscid Model," *Aeronautical Quarterly*, Vol. 33, Nov. 1982, pp. 293–312.

[3]Bryson, A. E., "Symmetric Vortex Separation on Circular Cylinders and Cones," *Journal of Applied Mechanics, Transactions of the ASME*, Vol. 81, Ser. E, 1959, pp. 643–648.

[4]Sarpkaya, T., "An Analytical Study of Separated Flow About Circular Cylinders," *Journal of Basic Engineering, ASME*, Vol. 90, 1968, pp. 511–520.

[5]Davis, M. D., "An Analytical Study of Separated Flow about a Circular Cylinder," M. Sc. Dissertation, Naval Postgraduate School, Monterey, CA, 1969.

[6]Hall, R. M., "Forebody and Missile Side Forces and the Time Analogy," AIAA Paper 87-0327, 1987.

[7]Sarpkaya, T., and Shoaff, R. L., "An Inviscid Model of Two-Dimensional Vortex Shedding for Transient and Asymptotically Steady Separated Flow Over a Cylinder," *AIAA Journal*, Vol. 17, No. 11, 1979, pp. 1193–1200.

[8]Almosnino, D., "High Angle-of-Attack Calculations of the Subsonic Vortex Flow on Slender Bodies," *AIAA Journal*, Vol. 23, No. 9, 1985, pp. 1150–1156.

[9]Sarpkaya, T., "Computational Methods with Vortices—The 1988 Freeman Scholar Lecture," *Journal of Fluids Engineering, Transactions of the ASME*, Vol. 111, No. 1, March 1989, pp. 5–52.

Reply by the Authors to T. Sarpkaya

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SARPKAYA'S comments on numerical instability associated with Bryson's vortex model are applicable only to a two-dimensional problem (a circular cylinder), and are not correct to the three-dimensional problems (cones and tangent ogives) we solved. The main differences between the two are the second term on the right side of Eq. (2), and the second term on the right side of Eq. (3) of Ref. 1. These terms play heavily on the force-free condition we used, $\boldsymbol{\Gamma} \times \mathbf{V} = 0$, where $\boldsymbol{\Gamma}$ is the vector of the line vortex strength. Therefore, Sarpkaya's statement that in our model, "the line vortex does not lie along a streamline ..." is totally incorrect.

In addition, since there is no force on the feeding sheet of vanishing small vorticity, there is no couple acting on the vortex in our model. The model does ignore the secondary separations.

In our model, the solution of each branch is very stable. In other words, if a solution is disturbed in any different way, the same final solution is always obtained. We are fully aware of the numerical problem associated with Bryson's model for a circular cylinder. This means that Bryson's model must be reformulated. However, this does not mean that the same numerical problem would occur in our three-dimensional solutions, since different models and equations are being used.

Reference

[1]Chin, S., Lan, C. E., and Gainer, T. G., "Calculation of Asymmetric Vortex Separation on Cones and Tangent Ogives Based on a Discrete Vortex Model," *AIAA Journal*, Vol. 27, No. 12, 1989, pp. 1824–1826.

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